Bohr Hydrogen Atom

Hydrogen Atom

\[ F_{(out)} = \frac{m v^2}{r} \quad ; \quad F_{(in)} = \frac{Zq q}{r^2} \]

Zq = nuclear charge, q = electron charge, Z is the # of Protons, and Z = 1 for Hydrogen Atom

\[ \frac{m v^2}{r} = F_{(out)} = F_{(in)} = \frac{Zq q}{r^2} \]

**Algebra**

\[ \frac{m v^2}{r} = \frac{Zq q}{r^2} \]

\[ \frac{1}{2} Mv^2 = \frac{Zq q}{2r} \]

\[ E_{(total)} = E_{(kinetic)} + E_{(Potential)} \]

\[ E_{(total)} = \frac{1}{2} Mv^2 + (-\frac{Zq q}{r}) = \frac{Zq q}{2r} + (-\frac{Zq q}{r}) = -\frac{Zq q}{2r} \]

\[ E_{(total)} = -\frac{Zq q}{2r} \]

to be used later
The Bohr Hypothesis is that the Angular Momentum of the Electron must be quantized:

\[ \text{Angular Momentum} = M \cdot V \cdot r = n \frac{h}{2\pi} \]

where \( n = 1, 2, 3, 4, \ldots \) is an integer value and is called the Bohr Quantum Number.

\[ M \sqrt{\frac{Zq q}{Mr}} \cdot r = n \frac{h}{2\pi} \]

Then just solve for \( r \).

**Result #1**

\[ r = n^2 \frac{h^2}{4\pi^2 M Zq^2} = \frac{n^2 a_0}{Z} = \]

\[ \frac{n^2 	imes 0.528 \text{ Å}}{Z} \]

\( r \) (radius) = 1 x 0.528 Å = 0.528 Å for \( n = 1 \)

= 4 x 0.528 Å = 2.110 Å for \( n = 2 \)

= 9 x 0.528 Å = 4.752 Å for \( n = 3 \)

= 16 x 0.528 Å = 8.448 Å for \( n = 4 \)

Only certain positions are allowed for the electron; the Electron Position is Quantized.

Note that

\[ E(\text{total}) = -\frac{Zq q}{2r}, \text{ an earlier result} \]

So
Result #2

\[ E(\text{total}) = - \frac{2 \pi^2 M Z^2 q^4}{n^2 \hbar^2} = - \frac{Z^2 R}{n^2} = - \frac{z^2 B}{n^2} \]

where \( B \) or \( R = 2.18 \times 10^{-11} \) ergs per atom

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Two Typical Calculations: Transition Energy and Ionization Potential

1. \[ \Delta E = E_{\text{final state}} - E_{\text{initial state}} = \left( - \frac{B}{n_f^2} \right) - \left( - \frac{B}{n_i^2} \right) \]

Where \( B = 13.6 \text{ ev per atom} = 2.18 \times 10^{-11} \) ergs per atom = 313.6 kcal per mole

\[ \Delta E = \frac{B}{n_i^2} - \frac{B}{n_f^2} = B \left( \frac{1}{n_i^2} - \frac{1}{n_f^2} \right) = h\nu = \frac{hc}{\lambda} \]

2. Ionization Energy \( IE \) = \( E \) infinite distance - \( E \) initial state

for the electron of the electron
\[ IE = 0 - \left( - \frac{B^2}{n_i^2} \right) = 0 - E_{\text{initial state}} = \frac{B^2}{n_i^2} = -E_{\text{initial state}} \]