Football “Winners” Score

Contributed by:
Howard Whitston (modified from D. Knuth’s Stanford vs. Harvard football problem)

Keywords:
Graph Theory, algorithm discovery, programming

Problem Statement:
There are 6 football teams in your division. Unfortunately, this year each team played only 4 games due to the weather and to make things worse, your team was unable to play your arch-rivals. So the question is, how your team do if the scores for the rest of the games played are used to determine the final score. There are at least two ways of answering this question, namely: 1) Best score using minimum number of teams and 2) Best score using all teams. Of course, “best score” means your team wins the game. When Dr. Knuth demonstrates his best score for the game between Stanford and Harvard, Stanford wins by a score of over 700 points over Harvard!

Type:
In-class group experiment, computer lab, research problem

Prerequisite Knowledge:
Graph Theory, Dijkstra’s Algorithm

Learning Outcomes:
To reinforce Graph Theory and when one can’t use Dijkstra’s Algorithm.

Connections:
Algorithm design strategies, complexity classes (there are two solutions: $n^2$, $n \lg(n)$)

Pedagogical Notes:
This problem can be done after doing Dijkstra’s Algorithm. It isn’t a trivial problem.
Football “Winners” Score

Brief Solution:
Below is a table for six teams, named ’A’ to ’F’ where the row team score is first followed the column team score and a dash indicates no games played (of course your team can’t play itself). By inspection, one solution is “D over F”, “A over D” giving a difference of 5 points (A winning). Another solution is “D over F”, “B over D”, “A over B” giving a difference of 14 points (again A winning). Is there a better solution? And is there a systematic way of finding it. Dynamic Programming is one possibility (Start with team F and work backwards toward team A. Problem: it doesn’t necessarily find the “best” solution since we aren’t using all teams.) Suppose there were eight teams and five games played, does your solution allow you to use the same method?

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>--</td>
<td>14–7</td>
<td>28–3</td>
<td>21–20</td>
<td>14–10</td>
<td>--</td>
</tr>
<tr>
<td>B</td>
<td>7–14</td>
<td>--</td>
<td>14–3</td>
<td>17–14</td>
<td>--</td>
<td>14–35</td>
</tr>
<tr>
<td>C</td>
<td>3–28</td>
<td>3–14</td>
<td>--</td>
<td>--</td>
<td>7–10</td>
<td>3–35</td>
</tr>
<tr>
<td>D</td>
<td>20–21</td>
<td>14–17</td>
<td>--</td>
<td>--</td>
<td>35–10</td>
<td>28–24</td>
</tr>
<tr>
<td>E</td>
<td>10–14</td>
<td>--</td>
<td>10–7</td>
<td>10–35</td>
<td>--</td>
<td>21–35</td>
</tr>
<tr>
<td>F</td>
<td>--</td>
<td>35–14</td>
<td>35–3</td>
<td>24–28</td>
<td>35–21</td>
<td>--</td>
</tr>
</tbody>
</table>

Sources:
D. Knuth, “Stanford GraphBase”, Chapter 1.5, pages 18-20; Chapter 2.6, pages 48-49; Chapter 5, pages 222-233 (source code)

Complete Problem Statement:
See Knuth’s book (two pages plus for the write-up).