Can you make the geodesic dome?

A geodesic dome is a three dimensional structure that looks roughly spherical, but is made of smaller triangles.

Geodesic domes are very strong and light. There are many different designs. Suppose you want to use plastic pipe make a geodesic dome like the one in Figure 1. There are two different edge lengths. The ratio of the lengths of the dotted edges to solid edges is about 0.884. The drawing is not scaled.

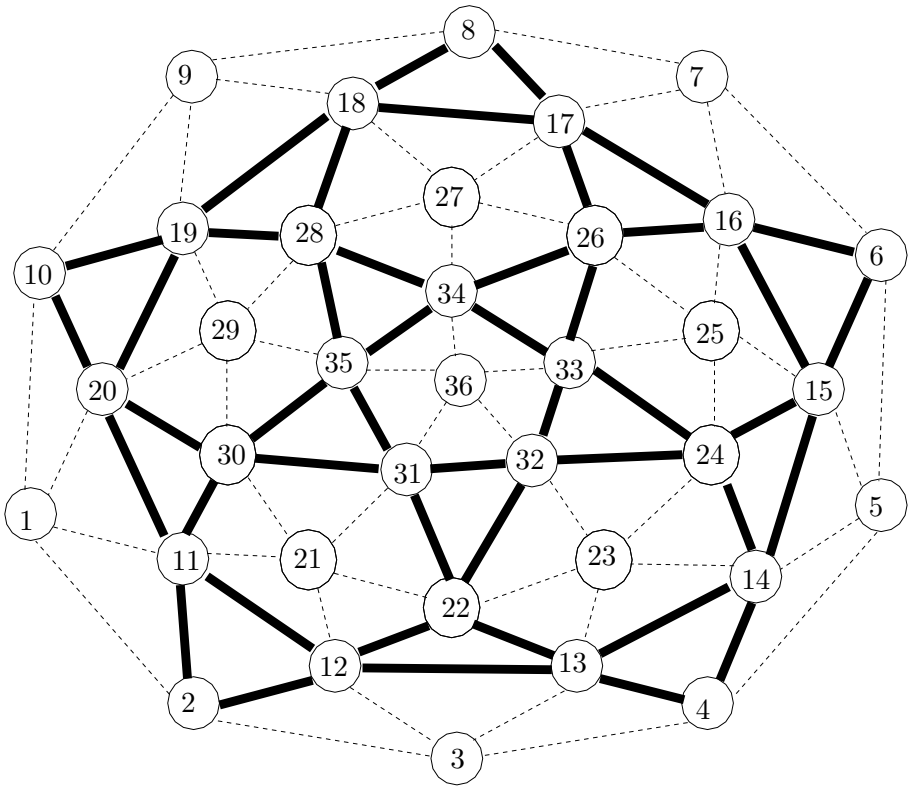
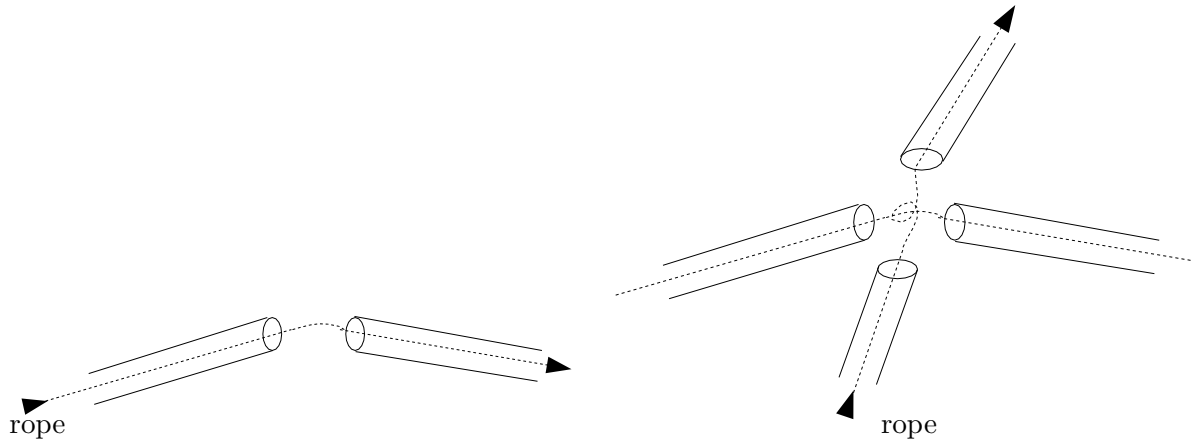


Figure 1: The design we will work with

The circles in Figure 1 represent the joints, at which the pipes must be fastened together. To join the pipes, you want to string a rope through each pipe, as shown in Figure 2. Note that the second and third times through the joint, the rope is looped around the pre-existing rope.

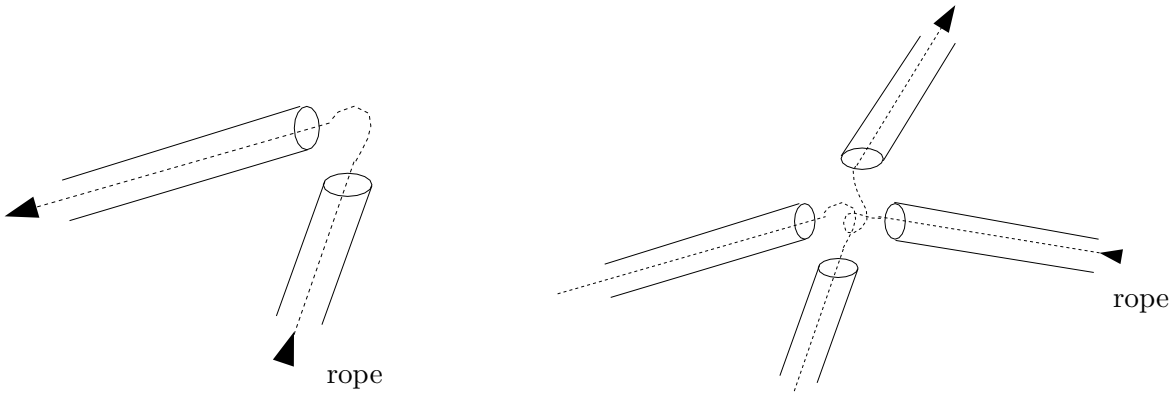


First time through a joint

Second time through a joint the new rope loops around the existing rope

Figure 2: How the rope forms the joints

The rope does not necessarily need to travel directly across a joint. It can take any other pipe out of the joint. See Figure 3.



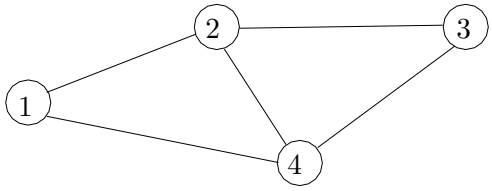
First time through a joint

Second time through a joint the new rope loops around the existing rope

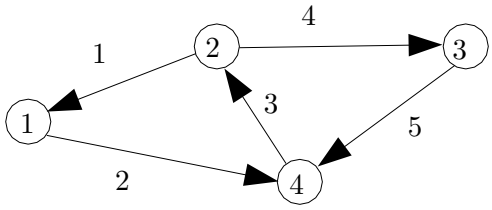
Figure 3: How the rope forms the joints

So all you need to do is to string the rope at least once through each pipe. You'd like to use the shortest rope possible. Your job will be to find out how much rope you need and what route the rope should take through the pipes.

Below is a small example of a different, two dimensional shape.



We could use the following route to string the rope exactly once through each pipe, starting at node 2:



If each of the pipes above are 2 feet long, then we would need a rope about 10 feet long. (We ignore the length required to do the looping at the joints.)

The following questions relate to the design in Figure 1.

1. Find the nodes of odd degree and list them.

2. Is there a way to string a single rope such that the rope goes through each pipe exactly once, with the ends of the rope at the *same* node? Explain your answer.

3. Is there a way to string a single rope such that the rope goes through each pipe exactly once, with the ends of the rope at *different* nodes? Explain your answer.

4. If the ends of the rope are at different nodes, what is the fewest number of pipes through which the rope must travel twice? Which nodes should have the rope ends?

5. Find a route through the pipes which minimizes the amount of rope needed. Draw it on Figure 1. (Bonus: There are many correct answers. Try to make yours easy to construct. Is it best to wander around or to focus on a single area first?)

Here is a picture of a geodesic dome made out of plastic pipes and held together with rope.