Answers to: Can you make the geodesic dome?

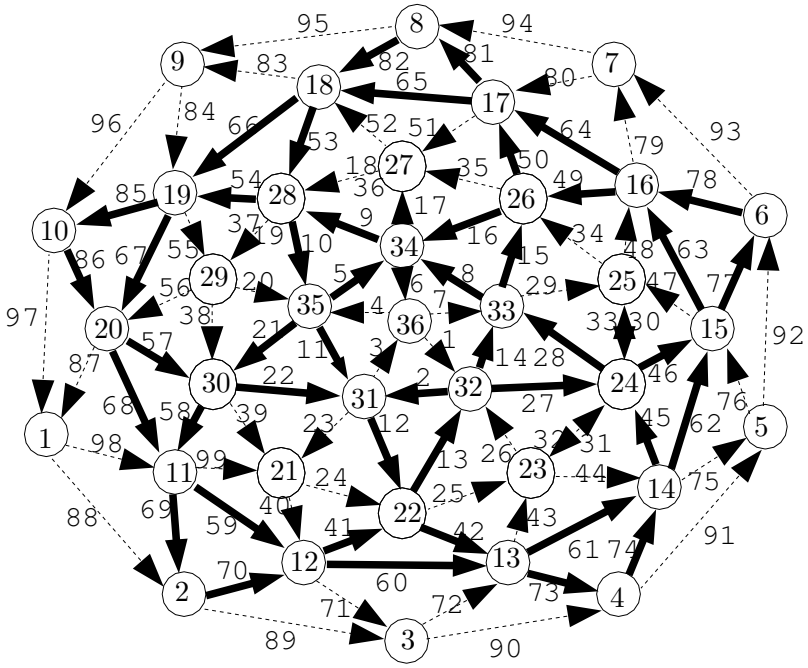
1. Find the nodes of odd degree and list them.
   There are six: 21, 23, 25, 27, 29, and 36.

2. Is there a way to string a single rope such that the rope goes through each pipe exactly once, with the ends of the rope at the *same* node? Explain your answer.
   No. This question is equivalent to asking if an Eulerian cycle exists on the graph in Figure 1. Since there are nodes of odd degree, Euler's theorem tells us that no Eulerian cycle exists.

3. Is there a way to string a single rope such that the rope goes through each pipe exactly once, with the ends of the rope at *different* nodes? Explain your answer.
   No. This question is equivalent to asking if an Eulerian trail (also goes by other names) exists on the graph in Figure 1. Since there are not exactly two nodes of odd degree (but rather six), Euler's theorem tells us that no such trail exists.

4. If the ends of the rope are at different nodes, what is the fewest number of pipes through which the rope must travel twice? Which nodes should have the rope ends?
   If we use two of the odd degree nodes for the rope ends (that is, the start and end of the trail), then the other four odd degree nodes must be made even degree by making the rope go through some pipes twice. Suppose we use nodes 21 and 36 for the rope ends. Then we have rope go through pipes 29-28, 28-27, 25-24 and 24-23 twice each, so we must travel through four of the dotted (shorter) pipes twice each. There are many other ways to do this.

5. Find a route through the pipes which minimizes the amount of rope needed. Draw it on Figure 1. (Bonus: There are many correct answers. Try to make yours easy to construct. Is it best to wander around or to focus on a single area first?)
   One answer is shown below. It starts at node 36 and ends at node 21. It is a bit hard to read, so here is the order in which the rope visits the nodes: 36, 32, 31, 36, 35, 34, 36, 33, 34, 28, 35, 31, 22, 32, 33, 26, 34, 27, 28, 29, 35, 30, 31, 21, 22, 23, 32, 24, 33, 25, 24, 23, 24, 25, 26, 27, 28, 29, 30, 21, 12, 22, 13, 23, 14, 24, 15, 25, 16, 26, 17, 27, 18, 28, 19, 29, 20, 30, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 11, 2, 12, 3, 13, 4, 14, 5, 15, 6, 16, 7, 17, 8, 18, 9, 19, 10, 20, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 1, 11, 21. Note that the upper part of the geodesic dome is constructed first in this answer, with each successively lower layer added in order.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